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Research

Modelling the evolution and diversity of cumulative culture

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Previous work on mathematical models of cultural evolution has mainly focused on the diffusion of simple cultural elements. However, a characteristic feature of human cultural evolution is the seemingly limitless appearance of new and increasingly complex cultural elements. Here, we develop a general modelling framework to study such cumulative processes, in which we assume that the appearance and disappearance of cultural elements are stochastic events that depend on the current state of culture. Five scenarios are explored: evolution of independent cultural elements, stepwise modification of elements, differentiation or combination of elements and systems of cultural elements. As one application of our framework, we study the evolution of cultural diversity (in time as well as between groups).

Keywords: cultural evolution; cumulative culture; mathematical model; cultural diversity; cultural systems

1. INTRODUCTION

In social science and the humanities, there is a long tradition of describing complex culture [1–4], and of exploring the evolution of culture and cultural systems [5–10]. An important observation about human culture is that it evolves in a seemingly open-ended manner which, among other things, includes the potential for the appearance of cultural elements of increasing complexity and refinement, and to form systems of culture [6,10]. Our aim in this theoretical paper is to develop a theory of cultural accumulation and explore how the cumulative aspect of culture may contribute to cultural diversity. To achieve this aim we will conceptualize cumulative culture in a way that lends itself to mathematical formalization.

The basic units of our theory will be *cultural elements* and *dependencies* between such elements. By a cultural element we here mean anything that may or may not be present in a given human society at a given time, like a tool or artefact, a method, an idea, a piece of knowledge. Dependencies refer to relationships between elements, such that the presence of one cultural element affects the likelihood that another element appears or disappears.

Dependencies between cultural elements seem to us to constitute the core of what cumulative culture is about. Other conceptualizations of cumulative culture in the literature seem to be special cases that are too limited. For instance, it has been proposed that cumulative culture is a culture that cannot be created within a single generation [11,12], which excludes cultural progress that proceeds in several steps within a single generation, and also begs the question *why* some culture cannot be created at such a high speed. Another example is the metaphor of cumulative culture as a ratchet [13] or as an irreversible process [10,14], which excludes the possibility that a cultural element can be lost and creates a one-dimensional image of progress. Dependencies between cultural elements, on the other hand, may describe not only refinement or progress, but also differentiation, combination of elements, substitutability (different solutions to the same problem), loss, and so on. Indeed, we propose that such 'multi-dimensional' aspects of cumulative culture are at the root of cultural diversity between societies. Specifically, we claim that accumulation of culture leads to diversity only if there are rich branching possibilities (differentiation and combination) or mutually inhibiting cultural elements, such that if one is present it inhibits the appearance of the other. We discuss these aspects in detail below.

Isaac Newton's famous statement that 'If I have seen further it is only by standing on the shoulders of giants' [15] could serve as a motto for our theory of cumulative cultural evolution. The general implication of this motto is that in order for human culture (of which science and mathematics are prime examples) to become increasingly sophisticated, it is *not* necessary that increasingly intelligent and creative individuals are born [16]. As knowledge, methods, ideas and tools cumulate, a new innovator who has no more talent than his predecessors can still come up with an innovation that is superior to what any predecessors have produced, precisely because he already has access to the contributions these predecessors

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One contribution of 14 to a Theme Issue 'Evolution and human behavioural diversity'.

made. In our theory we conceptualize this in probabilistic terms; a certain cultural element x may be very unlikely to appear, or even impossible, without the presence of some other cultural element y; when y appears (for whatever reason), the subsequent appearance of x may suddenly become highly likely.

The individuals of society are not explicitly represented in our models, despite the obvious fact that human agency is central in developing and transmitting culture. The virtue of an agent-less model is of course its simplicity, and the justification for making this simplification lies in Newton's statement. Although Newton was an exceptional scientist and mathematician, there is no reason to believe that the development of, say, differential calculus would not have happened without him; indeed, building on Descartes' work on analytical geometry, Newton's contemporary Gottfried Wilhelm Leibniz independently came up with an equally good solution to the problem of developing what is known as differential and integral calculus. To put it in general terms, the reason an agent-less model makes sense is that the innovations made in a society at a given time seem to be related much more strongly to the society's current culture than to its specific individuals. The framework we are proposing could be extended to include individual agents and thereby account for the degree to which a cultural element is established in a society (see §8). However, for the qualitative phenomena that we will discuss, it is sufficient to consider cultural elements as either present or absent in the society.

With these ambitions and limitations, our models are very different from the previous work on modelling cultural evolution. Such work has typically focused on how a predefined set of cultural elements may compete and spread in a population [17–21]. A smaller number of models allow accumulation of new cultural variants, but only along a single dimension [19,22–25]. These models are all individual-based, and the object is often to explore coevolution of genes and culture [26–28]. Whereas coevolutionary models consider how culture both influences and is influenced by genetic evolution, we are here interested in the impact of culture on *its own* evolution.

In the following, we will give a precise formulation of our theoretical framework, and then present a series of models exploring how different types of dependencies lead to cumulative cultural evolution that results in very different levels of cultural diversity between societies.

2. A FRAMEWORK FOR DESCRIBING HOW THE PAST INFLUENCES THE FUTURE IN CUMULATIVE CULTURE

It seems to us that almost every part of modern human culture (words, constructions and artefacts, behaviours, social organization, etc.) would have been alien to prehistoric humans. This illustrates that most cultural elements are not part of any ready-made human repertoire but instead come into existence in a particular population at some particular point in time. Here we will use the term 'appearance' for all such events, by which we mean that a cultural element is established in the population to the extent that it can influence further cultural change. Thus, we take a macroscopic approach in which we consider cultural elements that are present at the level of a cultural group (the framework can be extended to individual-based models by tracking the state of each individual rather than the population as a whole; cf. [29]).

The framework we develop here has two main features: it describes cultural evolution as (i) a stochastic process that (ii) at any point of time is partially dependent on the current cultural state. To say that a process is stochastic means that events occur with some probability, thus capturing the notion that cultural evolution is not completely predictable. Nor is cultural evolution completely unpredictable; the more we know about the current state of the world, the more accurately we can predict the coming events. In other words, future cultural evolution is not independent of the past.

There are many ways in which a current cultural element, say y, can influence the evolution of another cultural element, say x. For instance, if x can be obtained as a modification of y (possibly in combination with other cultural elements), then clearly x is much more likely to appear if the precursor y is present; the precursor could then either be replaced or remain and contribute to diversity. There are also more indirect types of influence, such as an element of general knowledge or an attitude that promotes or inhibits the appearance of x. An innovation may depend on more than just a single precursor and be subject to many indirect influences. This leads us to the notion that cultural elements often interact in larger *cultural systems*, which we will briefly explore in one of our models.

As a first step towards gaining a general understanding of processes of cumulation of complex culture, we will ignore the precise nature of the relationships between different cultural elements and only consider, in the abstract, how the probabilities of appearances and disappearances change when new elements appear or current elements disappear. From this perspective, there are only three ways in which y can influence the appearance of an element x that is currently not present: y may either *facilitate* or *inhibit* the appearance of x, or have no effect at all, i.e. be neutral or *independent*. These basic influences are described and exemplified in table 1. Disappearances of cultural elements follow the same logic: if x is already present then its disappearance may be either promoted or inhibited by y, or be independent of y.

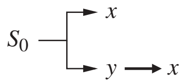
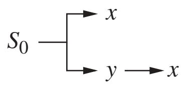
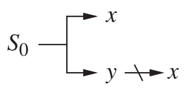
The probability of x appearing may of course depend on more than one other element. Indeed, it may depend on a large number of cultural elements as well as the current natural environment. To make possible a general description of such multiple dependencies, we will use the concept of a *current state of the world*. We will usually use the symbol S to denote the current state of the world.

(a) *Conditional probability functions of appearance and disappearance*

As a means to capture the totality of dependencies of future culture on the present culture, we introduce the following pair of mathematical objects:

$$\Pr(+x|S) \quad \text{and} \quad \Pr(-x|S).$$

Table 1. Kinds of dependencies of a cultural element, x, upon another cultural element, y. x, y, cultural elements; S_0, culture state without x and y; thicker lines indicate higher probability of transition.

dependence	histories	examples
facilitation		y is a tool, material or knowledge necessary to create x x is a modification of y x is a combination of y and another element (e.g. the harpoon combines spear and rope) y is a social institution that promotes x y is a technology that makes x cheaper
neutral		y is wholly unrelated to x
inhibition		y is a taboo that forbids x y is an alternative to x, e.g. a solution to the same problem

These are mathematical functions that for any cultural element x and any state of the world S return the conditional probability that x will appear, respectively, disappear (within some given short time period) given that the world is in state S. We will presently discuss how to mathematically represent the state of the world.

We want to emphasize that these probability functions can account not only for any dependencies of the current state of the world but also for other aspects that affect appearance and disappearance, such as cultural transmission and functionality. For instance, ceteris paribus, a cultural element that is difficult for individuals to acquire would have a lower appearance probability and a higher disappearance probability; similarly, a useful cultural element would typically have a higher appearance probability and a lower disappearance probability than a similar but useless element.

(b) *Starting conditions and cultural seeds*

A special case to consider is the situation where there exists no culture yet. It seems to us that almost all cultural elements are such that they can arise only in the presence of other cultural elements. In other words, only a very limited set of cultural elements are such that they can possibly appear from a situation where there is no previous culture. We will call such elements *cultural seeds*. In our models we will always assume that there is a limited number m of possible cultural seeds. The starting conditions of the cumulative process are defined by the set of cultural seeds and the probabilities for their respective appearances. These conditions will obviously affect the start of the cumulative process; for instance, higher probabilities of appearance of cultural seeds will lead to faster initial accumulation. However, initial conditions may also influence long-term evolution, for instance, by favouring one cultural system over another through path dependence. (We will discuss this further in §7.)

(c) *Representing the state of the world*

In principle, the variable S in the conditional probability function should specify every single detail of the world that can potentially affect the probability of appearance or disappearance of a cultural element x. This might include cultural factors as well as genetic and environmental factors. In practice, though, we need to specify models where S captures only the most relevant aspects of the world. In the present paper we will only deal with cultural states, but it would be just as easy to incorporate genetic or environmental influences (e.g. if one wanted to model that iron-based artefacts are more likely to appear in an environment where iron ore is easily accessible).

To describe cultural states we shall use the formalism of set theory. A state is defined as a set of elements (listed within curly brackets), so that for instance $S = \{y, z\}$ means that S consists of elements y and z. States change when new elements are added or old elements are removed. We will denote the addition of a single element x to a set S by $S + x$. Thus, for the example above we would have $S + x = \{x, y, z\}$. Similarly, we will denote the removal of an element by the minus sign, so that in our example we would have $S - y = \{z\}$. The number of elements in a set S is denoted by $|S|$, so in our example we have $|S| = 2$.

(d) *An example of complex cumulative cultural evolution*

As an example of how our framework might apply in practice, consider the proof of the Four-Colour Conjecture (4CC), a long-standing conjecture in mathematics stating that the regions of any map can be coloured using at most four colours so that no two regions sharing a border have the same colour [30]. The problem was first proposed by Francis Guthrie in 1842 and eventually gained wide attention among mathematicians. A proof of the conjecture was published by Alfred Kempe in 1879, but 11 years later it was shown, by Percy Heawood, to contain a crucial error. The conjecture withstood the continued attacks of many mathematicians for another century until it was proved by Kenneth Appel and Wolfgang Haken in 1977. By combining new ideas of their own with an idea developed in the 1960s by Heinrich Heesch (which itself built on Kempe's work), Appel and Haken succeeded in reducing the conjecture to a large but finite number of cases that were verified case-by-case using a computer. Thus, when Appel and Haken finally proved the 4CC they benefitted from several achievements by other people. In order to describe some core aspects of this instance of cultural evolution within our framework, we define four states of the world:

S_0 = the status of mathematics before 1842

$S_1 = S_0 + 4CC$

$S_2 = S_1 + \text{Heesch's idea}$

$S_3 = S_2 + S_2 + \text{computers}$

We claim that the probability of the appearance of a correct proof of the 4CC ought to increase as the state of the world changes from S_0 to S_3:

$$\Pr(+\text{proof}|S_0) < \Pr(+\text{proof}|S_1) < \Pr(+\text{proof}|S_2) < \Pr(+\text{proof}|S_3).$$

The interpretation of these inequalities is as follows. First, an innovation (the proof of 4CC) is more likely to happen if there is an explicit idea that such an innovation may be possible (the 4CC itself). Second, an innovation that is a combination of several parts is more likely if some part already exists (Heesch's idea). Third, a labour-intensive innovation is more likely if there exists adequate labour-saving tools (computers).

(e) *Combinations, components and facilitators*

The above example illustrates two main ways in which a pre-existing cultural element y can influence the likelihood of the arrival of a new cultural element x. One possibility is that the new cultural element x is a *combination* where y is a *component* (like Heesch's idea is a part of Appel and Haken's proof). To express that x is a combination of y and z we may write

$$x = y \circ z,$$

where $\circ$ denotes the operation by which the parts have been combined. The above example also shows how an element may facilitate the appearance of x without itself being part of x. The computers were necessary to carry out the proof but are not a component of the proof itself.

Generally, we will say that an element y facilitates the appearance of another element x if, for all relevant states of the world, it holds that $\Pr(+x|S+y) > \Pr(+x|S)$; similarly, y inhibits the appearance of x if $\Pr(+x|S+y) < \Pr(+x|S)$.

(f) *Representing dependencies between cultural elements as a graph*

Relationships between elements are often effectively illustrated using a graph with vertices and edges representing elements and relationships, respectively. We will draw edges with closed arrowheads for facilitating relationships, while open arrowheads signify inhibiting relationships; no edge means no relationship (i.e. independence).

The graphs in figure 1 represent some specific models that we will investigate. We have chosen these examples to illustrate a diversity of cases. To explore the extent to which cultures tend to diverge or remain similar, we define the similarity of two cultural states as the proportion of all elements present in either state that are shared by both states. See appendix A for details.

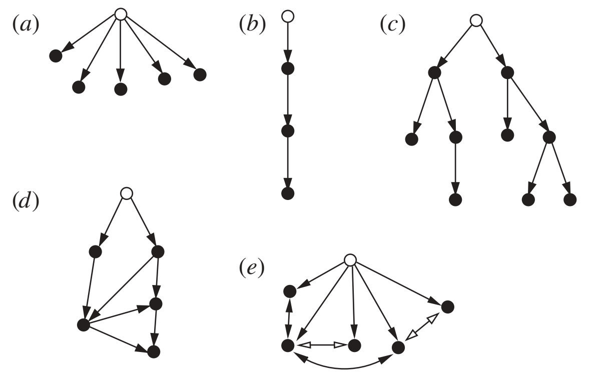


Figure 1. Examples of dependencies between cultural elements: (*a*) independent elements; (*b*) linear succession of elements; (*c*) differentiation of elements; (*d*) pairwise combinations of elements; (*e*) systems of cultural elements (open arrowheads represent inhibitory relationships). The open circle represents a state in which no culture is present.

3. INDEPENDENT CULTURAL ELEMENTS

A number of recent models have studied the evolution of independent cultural elements (figure 1*a*; [29,31–34]). Although these models allow accumulation of elements over time, they are not strictly models of cumulative culture in the sense of this paper because the appearance of new elements is neither facilitated (nor inhibited) by the existence of other cultural elements. In our framework, this means that the probabilities of appearance and disappearance of a cultural element are constants:

$$\Pr(+x|x \notin S) = q_{\text{app}} \tag{3.1}$$

and

$$\Pr(-x|x \in S) = q_{\text{dis.}} \tag{3.2}$$

From the point of view of our framework, this is the baseline case where there are no other possible elements than the cultural seeds themselves (i.e. all elements are accessible from a culture-less state). Throughout this paper we denote the number of cultural seeds by m. Hence, in this model there are only m elements that are at all possible.

Let n_t denote the expected number of elements present at time t. The expected number of elements present at time $t+1$ can be computed as

$$n_{t+1} = (1 - q_{\text{dis}})n_t + q_{\text{app}}(m - n_t), \tag{3.3}$$

where the first term accounts for the disappearance of a fraction q_{dis} of elements that exist at time t, and the second term is the expected number of elements that appear, out of the $m - n_t$ that do not exist at time t. Assuming that there are no elements at time 0, we have $n_0 = 0$ and equation (3.3) has the unique solution

$$n_t = \frac{mq_{\text{app}}}{q_{\text{app}} + q_{\text{dis}}}(1 - (1 - q_{\text{app}} - q_{\text{dis}})^t). \tag{3.4}$$

As t grows, the number of elements approaches the equilibrium value

$$n_\infty = \frac{mq_{\text{app}}}{q_{\text{app}} + q_{\text{dis}}}. \tag{3.5}$$

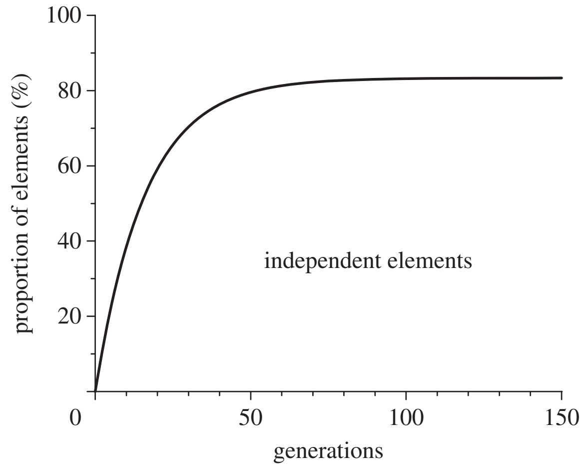


Figure 2. The expected path of cultural evolution (equation (3.4)) when all possible cultural elements appear and disappear independent of each other. Parameter values: $q_{\text{app}} = 0.05$; $q_{\text{dis}} = 0.01$.

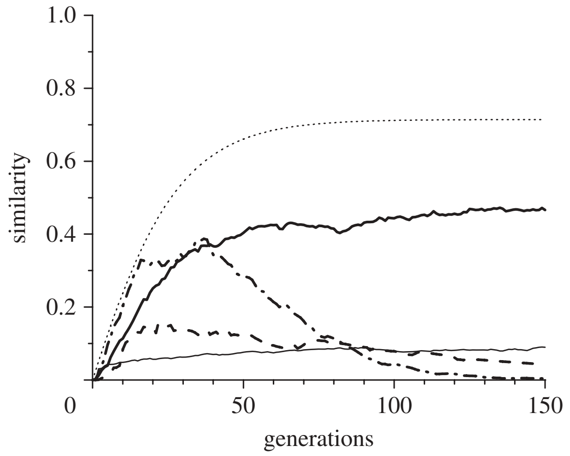


Figure 3. Average similarity between two independently evolved cultures according to models in the text. To allow comparison between different models, we chose a number $m = 2$ of cultural seeds in all cases, with the exception of the model of cultural systems in which we considered $M = 100$ possible elements. Similarity is calculated analytically in the case of independent elements (see appendix A), and as an average of the similarity observed in pairs of independent simulations in other cases (modifications: 500 simulations; differentiation, combinations and cultural systems: 100 simulations). Parameter values: $q_{\text{app}} = 0.05$; $q_{\text{dis}} = 0.01$; $m = 2$. Dotted lines, independent elements; solid thick line, modification; dashed line, differentiation; dashed–dotted line, combinations; solid thin line, cultural system.

Figure 2 illustrates how the expected number of elements varies over time. Although the expected number develops smoothly from 0 to the equilibrium value, simulated evolutionary trajectories present chance fluctuations in both the number of elements and which elements are present at a given time. Nevertheless, if the probability of appearances is significantly higher than the probability of disappearances, then the equilibrium number of elements will be close to m (saturation) and hence any two cultures will tend to become quite similar (figure 3). Indeed, assuming that both cultures are characterized by the same two parameters, $q_{\text{app}} \gg q_{\text{dis}}$, we can obtain an explicit solution. In this case, the formula for the expected cultural similarity at time t, (A 3) in appendix A, simplifies to

$$\text{expsim}(X_t, Y_t) = \frac{(n_t/m)^2}{2(n_t/m) - (n_t/m)^2} = \frac{n_t}{2m - n_t},$$

which at equilibrium takes the value

$$\text{expsim}(X_\infty, Y_\infty) = \frac{q_{\text{app}}}{q_{\text{app}} + 2q_{\text{dis}}}. \tag{3.6}$$

Here we see clearly that the expected cultural similarity is close to 1 if $q_{\text{app}} \gg q_{\text{dis}}$. The same saturation phenomenon also implies that culture will be rather static, as typically almost all m possible elements will be present at any given time once equilibrium is reached.

4. STEPWISE MODIFICATION

There are few formal studies that consider culture as more than a collection of independent elements. To our knowledge, the only relatively well-studied case is that of an ordered succession of elements, representing successive modification of an ancestral element (figure 1*b*). For example, tools such as hammers may be arranged in a succession of increasing functionality [6]. Element 0 of the succession would describe lack of hammers, element 1 very crude hammers such as unmodified stones, and so on. Alternatively, a succession can represent the stepwise development of a one-dimensional quantitative trait (e.g. the length of a spear), which is most efficient at some particular value [19,22]. We here consider the former case for illustration. Formally, we write x_i for the element at position i, and we assume that it can appear if the preceding element x_{i-1} exists:

$$\Pr(+x_i | x_i \notin S) = \begin{cases} q_{\text{app}} & \text{if } x_{i-1} \in S \\ 0 & \text{otherwise.} \end{cases} \tag{4.1}$$

As above, each element has an independent probability q_{dis} of disappearing. In this model, progress along the dimension is linear in time, provided $q_{\text{app}} > q_{\text{dis}}$ (figure 4*a*). This is the only possible path of cultural evolution, hence this model also produces considerable similarity between independently evolved cultures (figure 3).

It is easy to modify the model so that at any given time, the best of all present variants have a lower disappearance probability than inferior variants, which might be more realistic. However, in simulations we have found that such a modification does not lead to any qualitative change in results.

5. DIFFERENTIATION

In models of culture as an ordered succession, each element can be elaborated upon along a single dimension only. Typically, however, cultural elements can be modified in many ways. Hammers, for instance, can be specialized to serve different purposes, with an increase in the diversity of hammer types as well as

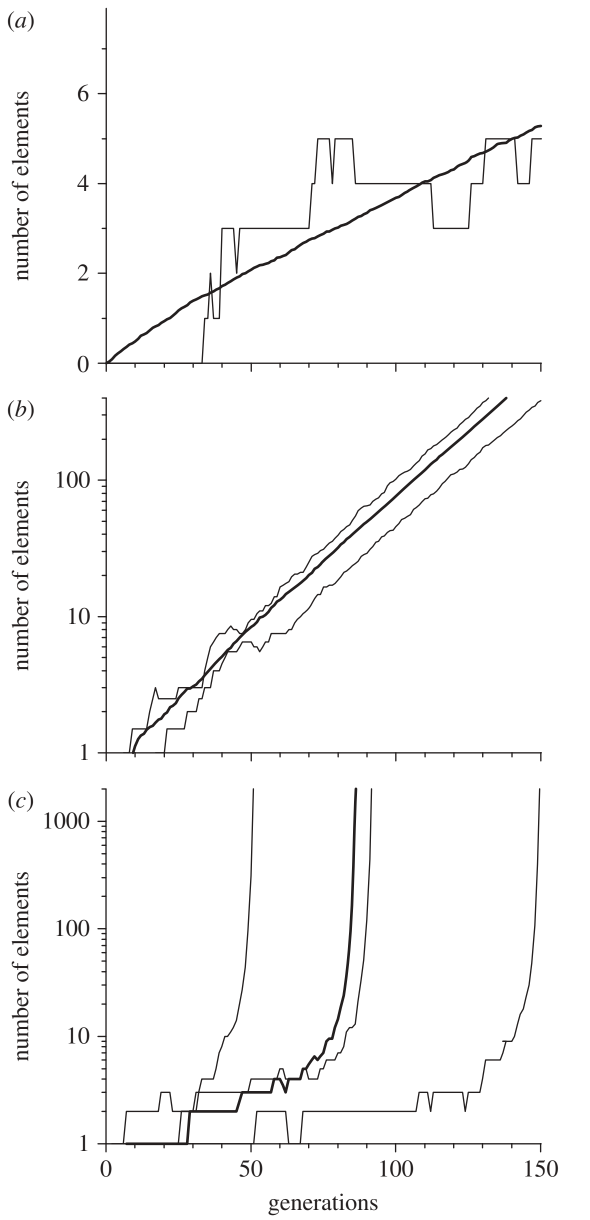


Figure 4. The expected path of cultural evolution, together with sample paths from individual simulations, for each of three models of cumulative cultural evolution. Parameter values for all models: $q_{app} = 0.05$; $q_{dis} = 0.01$. (*a*) Cultural evolution by successive modifications of elements. The number of seeds is $m = 1$. The average path is computed over 500 simulations (thick line, average; thin line, example). (*b*) Cultural evolution by differentiation of elements. The number of seeds is $m = 2$. Each element can differentiate into two elements. The average path is computed over 100 simulations (thick line, average; thin line, example). (*c*) Cultural evolution by pairwise combinations of elements. The number of seeds is $m = 2$. The median path is calculated over 100 simulations (thick line, median; thin line, example).

an increase in the efficiency of each type. Such an increase in diversity can be referred to as cultural differentiation.

A culture that evolves by differentiation of existing elements can be represented as a branching tree that originates from one of the cultural seeds (figure 1*c*). At any time during the differentiation process, any element can potentially differentiate into one or more new versions. Here we assume that all such differentiation events occur independently of each other with probability q_{app}. Thus, any element x that is not a cultural seed has a unique (direct) predecessor, $\pi(x)$. Only elements that are currently present can differentiate, so similar to the previous model of successive modifications we have

$$\Pr(+x|x \notin S) = \begin{cases} q_{app} & \text{if } \pi(x) \in S \\ 0 & \text{otherwise.} \end{cases} \quad (5.1)$$

The difference from the previous model is that an element may become the direct predecessor of more than one new element. In our simulations, we have set the number of potential successors to two. Under our usual assumption that each element has an independent probability q_{dis} of disappearing, the expected path of cultural evolution is exponential growth in the number of elements, provided that q_{app} is sufficiently larger than q_{dis} (see [34] for a related model and empirical examples of exponential cultural growth). An interesting observation is that individual runs of the model show considerable variation in the time of onset of accumulation but thereafter grow in a quite regular manner (figure 4*b*). Exactly which elements appear among the many possible choices, however, is a matter of chance. This results in little similarity between independently evolved cultures (figure 3).

6. COMBINATIONS

We now consider a model in which new cultural elements can be formed by a combination of the existing elements. For instance, a food y and a spice z can be physically combined to produce a dish $x = y \circ z$. However, by combination we also refer more broadly to any cultural element that can arise only if two (or more) component elements are present (such as the combination of mathematical ideas with computer technology in the the proof of the 4CC, as discussed earlier).

For simplicity, we assume here that only pairwise combinations can be formed, and that any two elements can be combined in only one way ($y \circ z = z \circ y$). However, we will assume that the order of successive combinations is crucial for the result. For example, cooking foodstuff y with technique z and then adding foodstuff w results in the dish $(y \circ z) \circ w$, which is generally different from the dish $(y \circ w) \circ z$ obtained by first combining the raw foodstuffs y and w and then cooking with technique z.

As usual, we will assume that there are m cultural seeds that can be invented directly from a cultureless state. Any other element can only be formed as a combination of two other elements, and can appear only if both components are present:

$$\Pr(+y \circ z|y \circ z \neq S) = \begin{cases} q_{app} & \text{if } y \in S \text{ and } z \in S \\ 0 & \text{otherwise.} \end{cases} \quad (6.1)$$

As in our previous models, we assume a constant disappearance probability of q_{dis}.

Cultural growth in this model is very fast, even faster than the exponential growth we saw in the model of cultural differentiation. The reason is that the number of elements that can be invented by pairwise combinations of n elements is of the order of n^2, while the number of elements that can be invented by differentiation is proportional to n.

Figure 4*c* shows the expected number of elements, computed as an average over 100 simulations. Similar to the case of differentiation above, in individual runs of the model, growth is initially erratic but becomes very regular after a few elements have appeared. At this stage, losses become negligible compared with the very high number of innovations that can appear by combining elements.

Figure 3 illustrates the average similarity of two independently evolving cultures. During the initial stages of growth, expected similarity increases owing to the relatively high probability of the cultures inventing the same cultural seeds and some of the simplest combinations. After growth picks up, however, the likelihood that the two cultures invent the same complex combinations is very small, hence similarity between cultures tends to drop quickly towards zero.

Finally, figure 5*b* illustrates the growth in complexity, defined as the number of evolutionary events (creations of cultural seeds and combinations of elements) necessary to create a certain cultural element. Thus, the complexity of a cultural seed is 1, the complexity of $y \circ z$ is 3, etc. As shown in the figure, the average complexity increases rapidly.

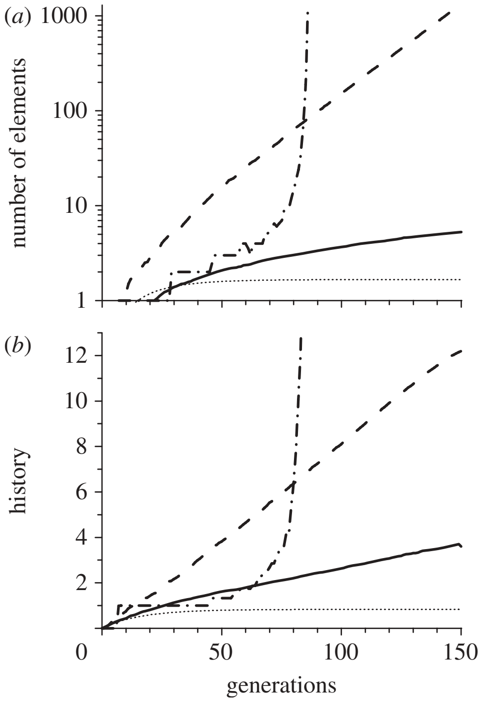


Figure 5. Comparison between the models of cumulative culture discussed in the text. (*a*) The number of elements in the culture (the vertical axis is logarithmic). (*b*) Average history of elements. The 'history' of an element is the number of evolutionary events that created the element, starting from a cultural seed. A value of 1 means that the element is a cultural seed, which evolved independently of other elements. Dotted lines, independent; solid line, modification; dashed line, differentiation; dashed–dotted line, combination. Parameter values as in figure 4.

7. CULTURAL SYSTEMS

In all previous models, the appearance of a given cultural element has depended on at most one or two others. In this section, we consider the cultural evolution of 'systems' of culture, in the sense of sets of interdependent cultural elements.

We first consider a model in which the probability that a cultural element appears depends on all elements in the culture. Here we assume that there is a set of N potential cultural elements, each of which may stand in either a *facilitating* or *inhibiting* relationship with any other element (as discussed in §2). For instance, a technology for melting iron ore may facilitate the appearance of iron tools; the practice of keeping an animal species for companionship may inhibit consumption of its meat.

To construct a simple model, let us say that an element x is *inhibited in cultural state S* if S contains more inhibitors than facilitators of x, and assume that x can appear in state S only if it is not inhibited in this state:

$$\Pr(+x|S) = \begin{cases} q_{\text{app}} & \text{if } x \text{ is not inhibited in } S \\ 0 & \text{if } x \text{ is inhibited in } S. \end{cases} \quad (7.1)$$

We also assume, as usual, that an element has a fixed probability q_{dis} of disappearing at each time step. The outcome of this model depends on the probability that elements facilitate or inhibit each other. When few inhibiting dependencies exist, most elements can appear in most cultural states, resulting in cultures with most of the m possible elements being present. In turn, this results in a high level of similarity between independently evolved cultures (figure 8). When the probability of inhibiting dependencies increases, cultures evolve to contain a smaller number of elements, and consequently are more different from each other.

Dependencies between cultures can also, potentially, influence the disappearance of elements. For instance, we may assume that an element that is facilitated in state S cannot disappear from that state:

$$\Pr(-x|S) = \begin{cases} q_{\text{dis}} & \text{if } x \text{ is not facilitated in } S \\ 0 & \text{if } x \text{ is facilitated in } S. \end{cases} \quad (7.2)$$

An influence of cultural state on the disappearance of elements may give rise to new phenomena, among which rivalling systems and combinations of independent systems.

(a) *Rivalling systems*

Figure 6 shows the dependencies between eight cultural elements represented as a graph in which edges represent facilitating relationships, and absence of edges represents inhibiting relationships. In this graph, we can identify two cultural systems: $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8\}$. By this we mean that elements within A typically facilitate each other and inhibit elements outside A, and the same goes for B. Neither system, however, is perfectly free from conflict. For instance, within system A elements 3 and 4 inhibit each other and facilitate outsiders 5 and 6,

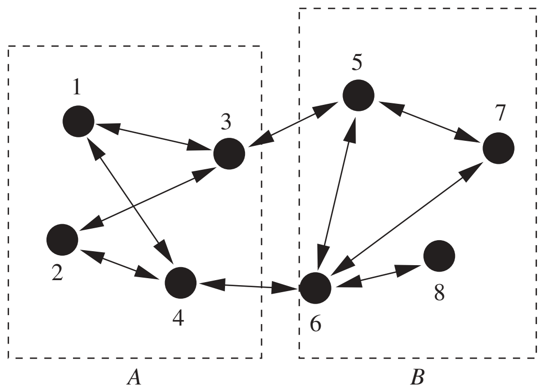


Figure 6. Example of a system of relationships between eight cultural elements. Edges indicate facilitation, missing edges inhibition. We can identify two cultural systems A and B, i.e. sets of cultural elements which, typically, facilitate each other and inhibit elements outside the set.

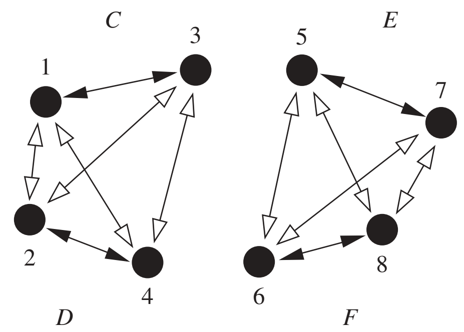


Figure 7. A system of relationships between eight cultural elements. Edges with closed arrowheads indicate facilitation, edges with open arrowheads indicate inhibition and missing edges indicate neutral relationships. Sets C and D are mutually exclusive (elements in C inhibit elements in D, and vice versa), as are systems E and F. Sets C and D are, however, compatible with sets E and F, as only neutral relationships exist.

respectively. Nevertheless, we expect cultural evolution to establish either system A or system B. Which of these two systems becomes established is a matter of chance events in the beginning of the evolutionary process.

In 1000 simulations of this example, using the appearance and disappearance rules equations (7.1) and (7.2), the culture ended up in system A 406 times and in system B 594 times. Thus, owing to the strong influence of cultural state on the appearance and disappearance of elements, the similarity between two independently simulated cultures was always either 0 or 1.

(b) *Combinations of independent systems*

When there are also neutral relationships between elements, collections of several smaller cultural systems can emerge and coexist independently of each other. An example is given in figure 7. Here, there are four identifiable systems: $C = \{1, 3\}$, $D = \{2, 4\}$, $E = \{5, 7\}$ and $F = \{6, 8\}$. Systems C and D are mutually exclusive, as are systems E and F. However, the first two systems are independent of the second two. In this situation, we therefore expect any of the four possible combinations of systems (CE, CF, DE and DF) to become established with equal probability.

In summary, complex webs of positive and negative dependencies between potential cultural elements will give rise to emergent cultural systems or collections of systems. Characteristics of such systems are that: (i) they are highly path-dependent, so that different cultural groups may develop very different cultural systems despite the same initial potential for culture; and (ii) the systems consist of elements that are on the whole mutually supporting but where some conflict between elements may be unavoidable (e.g. elements 3 and 4 in system A in figure 6).

8. DISCUSSION

In this paper, we have developed a theoretical framework for exploring cumulative cultural evolution. It is based on the simple idea that the existing cultural elements can facilitate or inhibit the appearance of new elements as well as the disappearance of present elements (table 1). With many cultural elements, the set of interdependencies can become arbitrarily complex (e.g. norms about what can be eaten can influence farming, breeding, household practices, etc., and can be influenced by religion and other traditions), so that evolution can only be understood if the whole system is studied together.

By describing how different cultural states influence the appearance and disappearance of cultural elements, we can explore long-term cumulative cultural evolution as a succession of appearance and disappearance events. By studying a series of different scenarios, we have shown in this paper that the nature of dependencies between cultural elements has dramatic effects on the pattern of cultural evolution. For example, when any given element facilitates the appearance of several similar elements, we observe a process of cultural differentiation in which the number of elements grows exponentially in time. In contrast, if elements can vary only along a single dimension, we observe linear growth. We stress that both the description of dependencies between elements and the process of cultural evolution that emerges from such dependencies lend themselves naturally to mathematical formulation. Indeed, they would be very difficult to explore without mathematics.

The results we have derived here raise many questions for future work. Below, we first discuss our results on the evolution of cultural diversity, then we discuss some open issues.

(a) *The evolution of cultural diversity*

The expression ‘cultural diversity’ can refer to several phenomena. Thus in developing a theory of cultural diversity we can ask many distinct questions, such as:

(1) Why do different cultures exist, and what determines how many different cultures are there?
(2) Given that distinct cultures exist, what governs their similarities and differences?
(3) What determines the number of cultural elements within a culture?

Table 2. Effect of several factors on the number of cultural elements and on the differences between cultures.

factor	number of cultural elements	difference between cultures
branching possibilities	+	+
facilitation	+	−
inhibition	−	+

(4) What determines the diversity of cultural elements within a culture, in terms of similarities and differences between the elements?
(5) What determines the extent to which individuals within a cultural group carry the same or different cultural elements?

Here we have mainly considered questions (2) and (3). Our analysis has highlighted a number of factors, summarized in table 2, that influence both the number of elements in a culture and the extent to which two independently evolved cultures share common elements. The first factor is the number of branching possibilities, that is, the possibilities to create new cultural elements from the existing ones (e.g. by differentiation or combination). If there are plenty of such possibilities, we have seen that cultures tend to become larger and less similar to each other (figures 3 and 5). Figure 4 also shows that, when many innovations are possible, cultural elements accumulate longer *histories*, i.e. they arise from many evolutionary steps. Table 2 also points to the complementary influences of facilitation and inhibition on cultural diversity, the former promoting amount of culture, the latter promoting difference between cultures. Mutual facilitation makes it more likely that similar sets of elements eventually appear in different cultures, even if the cultures initially contain different elements. Mutual inhibition increases cultural differences because different cultures may establish different subsets of mutually incompatible elements (figure 8).

A third source of cultural diversity is chance. Because the appearance and disappearance of elements have stochastic components, we generally observe random variation both in the time of appearance of specific elements and in what elements actually appear. Some effects of chance can be appreciated by contrasting average paths of cultural evolution with single simulation runs in figure 4. When many possibilities for innovation exist, and in the presence of inhibitory dependencies between elements, chance is particularly important in choosing which of the many possible paths a particular culture actually takes. This means that any two cultures are unlikely to take exactly the same path (multilinear evolution) [10]. Even in the presence of random factors, however, there can be surprising regularities. One example is the regular growth in the amount of culture in the model of cultural differentiation, reminiscent of a steady rate of genetic change in genetic evolution, the 'molecular clock' metaphor [35].

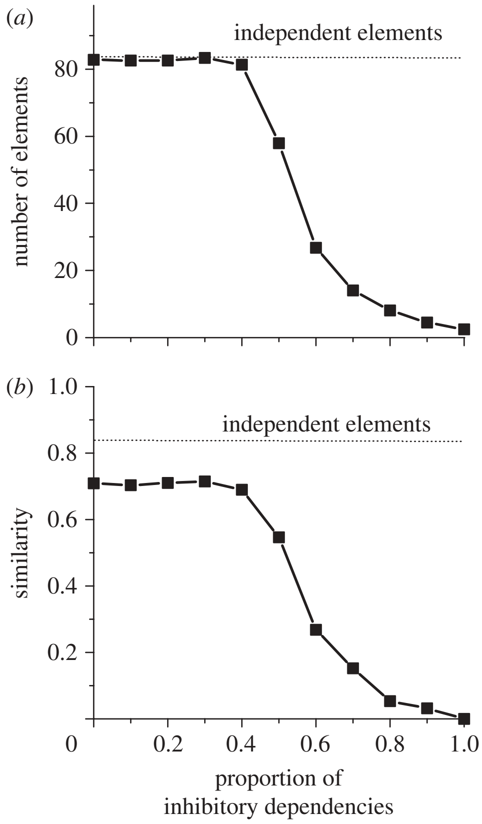


Figure 8. Simulation of cultural evolution when appearance of a cultural element depends on the number of facilitating versus inhibiting elements in the current cultural state (equation (7.1) with $q_{\text{dis}} = 0.05$), and disappearance of element is random with probability $q_{\text{dis}} = 0.01$). (*a*) Similarity of independently evolved culture as a function of the probability that the relationship between any two elements is inhibiting versus facilitating. (*b*) Size of evolved cultures under the same conditions. A number of $m = 100$ possible elements was considered.

It is possible to extend our framework to address questions (4), (5) and (1) above. To tackle question (4) we need to define a measure of similarity between cultural elements, e.g. similarity in function, appearance or history. Similarity measures based on different criteria may sometimes agree, though not always (cf. analogy and homology in genetic evolution) [35]. For instance, two hammers that are derived from the same, pre-existing hammer will often be similar in function, appearance and, of course, history. Once a measure of similarity between cultural elements is defined, it is possible to use it within our framework to study the similarity of evolved cultural elements. Whether this can be a fruitful line of research is a question for future work.

Question (5) concerns how culture is distributed among individuals in a group. Although we have not considered how individuals carry culture, it seems clear that maintaining a large culture requires individuals to specialize, i.e. each to carry a different subset of the culture (this follows from the simple fact that each individual has limited memory). Thus conditions that favour a large culture are also expected to foster within-group cultural diversity, i.e. specialization. We also point out that specialization itself may favour cultural growth, because if individuals do not need to carry all of the group's culture they may have more resources to create new culture within their specialization.

Question (1) is akin to asking how different biological species evolve, and what determines their number and abundance. Cultural diversity in this sense, therefore, may be quantified using measures of biodiversity [36]. A true understanding of how such cultural diversity evolves, however, requires significant theoretical developments, which lie beyond our present scope. Note that questions (1) and (5), while seemingly at opposite ends of a spectrum ranging individuals to cultures, may actually be strictly interrelated. The reason is that a complete understanding of how distinct cultures emerge requires the understanding of how cultural differences develop between individuals. Addressing questions (1) and (5) requires a refinement of our framework in which the appearance and disappearance of traits is tracked at the level of individuals [29,37].

(b) *Causes of the appearance and disappearance of cultural elements*

We have assumed in our models that cultural elements appear and disappear solely based on their dependencies on other elements. In reality, many other factors contribute. For instance, the appearance and disappearance of a cultural element are influenced by its functionality. For example, many combinations of cultural elements seem unlikely to appear simply because they can serve no function (think of the possible combinations of 'pasta', 'tomato sauce', 'hammer' and 'computer'). Introducing such cultural selection based on function may or may not change the general patterns of growth analysed above. Consider, for instance, the model of cultural accumulation along a single dimension. If some elements are much more efficient than others (e.g. a certain length for a spear), and if function is the main determinant of element's appearance and disappearance, we expect evolved cultures to consist mostly of the few efficient elements; we no longer expect the number of elements to grow linearly in time once the functional elements have appeared [19]. Some of our results, however, appear more robust. For example, consider the model of cultural differentiation and suppose that only a fraction of the elements that can be derived from any given element is functional. We still expect an exponential increase in the number of elements, albeit at a slower rate. Thus, a system in which cultural elements can differentiate would still produce more diversity than a system that develops along one dimension only, and less diversity than a system in which cultural elements can combine.

We have also left out the effect of environmental variation and genetic factors on cultural evolution. The environment can be incorporated in the 'state of the world' so that, for example, a cultural element may be more likely to appear in one environment than in another. Similarly, genetic predispositions may influence appearance and disappearance probabilities. For instance, facilitation and inhibition between cultural elements could be viewed as reflecting the impact of evolved mental structure. An extension of our framework to individuals would enable the study of the interplay between individual genetic variation and cumulative cultural evolution.

(c) *Cultural complexity*

The greatest challenge in studying the evolution of cultural diversity lies perhaps in the complexity of cultural systems. We have only touched upon this topic in our last model, but our framework can cover a wider range of cases where processes of refinement, differentiation, combination, facilitation and inhibition, which we have studied separately, occur simultaneously.

We believe that an advantage of our approach in the study of complex culture is a stronger focus on creativity and cultural history, compared with most current theory which emphasize social learning as the main force in cultural evolution [11,19,27]. Social learning (in a broad sense) can explain how culture is maintained in time (including why some elements may be more easily retained), but the most spectacular feature of human cultural evolution is the open-ended process of creation of novel, often increasingly complex culture. Although individual creativity has been the subject of much investigation [16,38], very little is clearly understood about how creativity shapes long-term cultural evolution. We believe that a framework like ours is helpful, possibly even necessary, for real progress to be made on this topic.

There are many steps left to be taken, for which our framework can be a starting point. For instance, the issue of the consequences of human intentionality could be explored through studies of the interaction between different *kinds* of cultural elements, such as ideas (about what is possible), opinions (about what is important) and goals (for what to achieve).

Another obvious route to go is to incorporate more fine-grained aspects about the population to make it possible to deal with issues like specialization and subcultures within groups, and interactions between cultural groups.

Our models also point to the importance of what we have called 'cultural seeds', i.e. cultural elements that can appear in the absence of pre-existing culture. It may very well be the case that there does not exist a very large set of cultural elements that are all essentially independent of each other and that can evolve from a situation without any culture. Theoretical and empirical explorations of this issue are, to our knowledge, extremely limited, with the possible exception of ideas within structural anthropology [39].

(d) *Conclusion*

We sought to capture in a clear formal framework what we believe is the essence of cultural accumulation: the unlimited potential for innovation and the complex dependencies between cultural elements. Our approach offers, to our knowledge for the first time, a way to model at least some of the complexity of cumulative cultural evolution beyond the ideas of a simple accumulation of elements or one-dimensional improvement. To be really useful, however, our approach must be connected productively to empirical observations of cultural dependencies, cultural evolution and cultural history. We believe that this is possible through investigations of actual trajectories of cultural evolution and studies of relationships between cultural elements. As an example of empirical data that are relevant here, it has been shown that the number of cultural elements in some domains has grown exponentially [34], suggesting that differentiation has been a major underlying process in these cases. One example of empirical studies we would like to see done is analyses of absence of particular elements in cultural systems in terms of presence of inhibiting elements.

There has so far been little common ground between mathematical theory of cultural change and mainstream work on cultural change in anthropology and other social sciences [10]. Our framework may help to strengthen the connection, as the evolving cultural systems presented here could be used to model many existing notions within the human sciences (e.g. within the fields of ethnicity, sex and gender, social norms, world views and subsistence systems) about how various ideas and practices may support or be in conflict with each other.

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APPENDIX A

To quantify cultural diversity, we define the similarity of two cultural states as the proportion of all elements present in either state that are shared by both states. Formally, if X and Y are the sets of elements representing the two states, and assuming that at least one is not empty, then their similarity is defined as

$$s(X,Y) = \frac{|X \cap Y|}{|X \cup Y|} = Pr(x \in X \cap Y | x \in X \cup Y), \quad \text{(A 1)}$$

where x is a random element drawn uniformly from the set of all possible elements (assumed to be finite). If the cultural states X and Y arise from a stochastic process, they are themselves random variables, and we define their expected similarity by

$$\text{expsim}(X,Y) = \text{E}(s(X,Y)|X \cup Y \neq \emptyset).$$

Then we have

$$\begin{aligned}\text{expsim}(X,Y) &= Pr(x \in X \cap Y | x \in X \cup Y) \\ &= \frac{Pr(x \in X \cap Y)}{Pr(x \in X \cup Y)}. \end{aligned} \quad \text{(A 2)}$$

Assuming that states X and Y have evolved independently of each other, the last expression can be rewritten as

$$\text{expsim}(X,Y) = \frac{Pr(x \in X)Pr(x \in Y)}{Pr(x \in X) + Pr(x \in Y) - Pr(x \in X)Pr(x \in Y)}. \quad \text{(A 3)}$$

Thus, we can calculate the expected similarity between cultures if we know the probability that an element is part of a cultural state.

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